Use Pascal's Triangle to expand and simplify $(2n^4 - r^3)^5$. **SCORE: _____ / 5 PTS**

$$(2n^4)^5 + 5(2n^4)^4(-r^3) + 10(2n^4)^3(-r^3)^2 + 10(2n^4)^2(-r^3)^3 + 5(2n^4)(-r^3)^4 + (-r^3)^5$$

$$= \underbrace{32n^{20}}_{\frac{1}{2}} \underbrace{-80n^{16}r^3}_{1} + \underbrace{80n^{12}r^6}_{1} \underbrace{-40n^8r^9}_{1} \underbrace{+10n^4r^{12}}_{1} \underbrace{-r^{15}}_{\frac{1}{2}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{310}(2n^2 - 5n + 3)$. **SCORE: _____ / 4 PTS**

Your final answer may **NOT** use … **NOR** $\sum$. It may use $+, -, \times, \div$. (It does **NOT** need to be simplified into a single number.)

$$2\sum_{n=1}^{310} n^2 - 5\sum_{n=1}^{310} n + \sum_{n=1}^{310} 3$$

$$= \underbrace{2 \cdot \frac{310(311)(621)}{6}}_{2} \underbrace{- 5 \cdot \frac{310(311)}{2}}_{1\frac{1}{2}} \underbrace{+ 310 \cdot 3}_{\frac{1}{2}}$$

FJ started a 31 day treatment program which involved daily injections of a medication. The first day's injection was 18 mg, and each subsequent day's injection was 6% less than the previous day's injection. Find the total amount of the injections. Your final answer may **NOT** use ... **NOR** $\sum$. It may use +, −, ×, ÷ and powers. (It does **NOT** need to be simplified into a single number.) **SCORE: _____ / 5 PTS**

a_n = AMOUNT OF INJECTION ON n^{TH} DAY

$a_1 = 18$

$a_2 = 18(1-0.06) = 18(0.94)$

$a_3 = 18(0.94)^2$

$\vdots$

$a_n = 18(0.94)^{n-1}$

1 2½

$$\text{TOTAL} = \frac{18(1-0.94^{31})}{1-0.94}$$

1½

Find the rational number representation of the repeating decimal $0.4\overline{27}$ using the method discussed in lecture. **SCORE: _____ / 6 PTS**
NOTE: Only the 27 is repeated.

$$0.4 + 0.027 + 0.00027 + 0.0000027 + \ldots \quad (1)$$

$$= \frac{4}{10} + \frac{0.027}{1 - \frac{1}{100}}$$

$$= \frac{4}{10} + \boxed{\frac{\frac{27}{1000}}{\frac{99}{100}}} \quad (2\tfrac{1}{2})$$

$$= \frac{4}{10} + \frac{\overset{3}{\cancel{27}}}{1000} \cdot \frac{100}{\underset{11}{\cancel{99}}}$$

$$= (\tfrac{1}{2})\ \boxed{\frac{4}{10}} + \boxed{\frac{3}{110}} \quad (1)$$

$$= \boxed{\frac{47}{110}} \quad (1)$$

Use mathematical induction to prove that is 3 a factor of $n^3 + 6n^2 + 8n$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: 3 IS A FACTOR OF $1^3+6(1)^2+8(1)=15$ (1)

MUST HAVE WORD "INTEGER" (1)

INDUCTIVE STEP: ASSUME 3 IS A FACTOR OF k^3+6k^2+8k (1)

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$ (1)

[PROVE 3 IS A FACTOR OF $(k+1)^3+6(k+1)^2+8(k+1)$]

$(k+1)^3+6(k+1)^2+8(k+1)$ (1)

$$= k^3+3k^2+3k+1$$
$$+6k^2+12k+6$$
$$+8k+8$$

$$=(k^3+6k^2+8k)+(3k^2+15k+15)$$

$$=(k^3+6k^2+8k)+3(k^2+5k+5)$$ (2)

SINCE 3 IS A FACTOR OF BOTH (k^3+6k^2+8k) AND $3(k^2+5k+5)$ (1)

THEREFORE 3 IS A FACTOR OF $(k+1)^3+6(k+1)^2+8(k+1)$ (1)

SO, 3 IS A FACTOR OF n^3+6n^2+8n ($\frac{1}{2}$)

FOR ALL INTEGERS $n \geq 1$. ($\frac{1}{2}$)